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# Open string gravity?

**Yeuk-Kwan E. Cheung, Mark Laidlaw and Konstantin Savvidy**

*Perimeter Institute for Theoretical Physics*
*35 King St. North, Waterloo, ON N2J 2W9, Canada*
*E-mail:* echeung@perimeterinstitute.ca, mlaidlaw@perimeterinstitute.ca, ksavvidis@perimeterinstitute.ca

Abstract: We present a new application of Boundary String Field Theory: calculating the induced-gravity action on a D-brane. Using a simple quadratic tachyon potential to model a D-brane fluctuating in the flat target space we derive the effective action in terms of the extrinsic curvature to all orders in $\alpha'$. We identify both the Born-Infeld structure as well as the Einstein-Hilbert term at order $\alpha'$. This corroborates the conjectured existence of the latter term in the brane-world scenarios.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, D-branes, String Field Theory.

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## 1. Introduction

The brane-world scenario [1] — confining degrees of freedom within a subspace inside a higher dimensional space — has been one of the interesting new ideas in theoretical physics in the past years. This scenario has been advocated in attempts to find solutions to the hierarchy problem, to explain the cosmological constant and dark matter, and to parametrize the matter content of our world. One theme that receives attention concerns the description of gravity on the brane, its localization properties and the effects of higher dimensional gravity. Many authors have introduced in a physically motivated manner the Hilbert-Einstein term and higher curvature invariants to the action in the brane world-volume. There is no universal agreement in the literature, however, about what corrections are appropriate to include. Due to the volume of work on this subject, the readers are referred to [1]–[6] for a sample of the original works and to [7, 8] for thorough reviews. In light of these exciting developments we derive this type of action directly from a fundamentally stringy approach. Several authors have taken a related but different approach, and work out the world-volume action by examining string amplitudes for scattering of gravitons into various world-volume fields [9, 10, 11, 12].

The dynamical degrees of freedom on the brane are those of the transverse excitations. These massless scalars, $\phi$, originate physically from the spontaneously broken translation symmetry of the position of the brane. In string theory these degrees of freedom are part of the massless spectrum of the open strings ending on the brane. Together with the other massless open strings which are the gauge fields they can be described by the Dirac-Born-Infeld (DBI) action [13]–[16]. The standard form of the DBI action incorporates the higher powers of the gauge field strength (curvature) in fact it does so to all orders in $\alpha'$. We observe that the corrections to the D-brane effective action arising from higher orders in the curvature of the brane world-volume are relatively much less explored. In this paper we develop a method to systematically compute in the framework of open string field theory those corrections to any given order in the extrinsic curvature of the brane world-volume. This expansion in power of $\alpha'$ like in the case of DBI, incorporates stringy effects of only worldsheets of the lowest genus, the disk.



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To an observer living on the worldvolume of the flat brane (embedded in flat ambient space) the transverse deviations principally manifest themselves as perturbations of the metric away from flat Minkowski metric.

$$G_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \phi \, \partial_\nu \phi \, .$$

These metric perturbations propagate at the speed of light and may conceivably be detected by an apparatus like LIGO. In this respect those metric perturbations are acting similarly to gravity. However we would like to stress some important differences:

- The number of dynamical degrees of freedom is equal to the co-dimension of the brane and is independent of the dimensionality of the worldvolume.
- The degrees of freedom of this induced gravity get their kinetic term not from the Einstein-Hilbert term, present also in this case, but from the brane tension.

These are in stark contrast to ordinary gravity, or indeed the brane-gravity approach where the entries of the metric tensor are the degrees of freedom and Einstein-Hilbert term provides the kinetic term for the graviton and the tension term would be interpreted as the cosmological constant. Ultimately, the question of whether or not the present theory reproduces the known facts about gravitational force is likely to hinge on the interaction of the metric perturbations with matter. On this front, there is the reassuring evidence that the photon field, representing the open-string gauge field degrees of freedom, does in fact have the required coupling through the energy-momentum tensor to the said metric perturbations.

In this letter we propose a simple technology, based on boundary string field theory, for the computation of the higher curvature corrections to the induced action on a D-brane. For the sake of simplicity we are setting $F = 0$ and so concern ourselves with only the induced metric degrees of freedom. To model a D-brane we insert a quadratic tachyon potential on the boundary of the string worldsheet to confine the endpoints of the open string. We obtain a closed expression to all order of $\alpha'$:

$$\mathcal{S} = \int dx_{||} \sqrt{\det(G_{\mu\nu})} \int dx_\perp \; e^{-x_\perp^2} \exp\left( \sum_j (-)^j \frac{\zeta(j)}{j} \operatorname{tr}(l_s \, K \cdot x_\perp)^j \right),$$

where $K$ is the extrinsic curvature.

Our approach has a number of advantages, the principal one being its calculational simplicity. A novel point is that we do not introduce the position of the brane under consideration as part of a gauge field, but rather encode it in the open string tachyon. This allows us to progress with considerably less effort past the Born-Infeld limit. As we shall explain below, we base our analysis only on the properties of the quadratic interactions on the boundaries of an open string worldsheet. We exhibit our method for the bosonic string but it can easily be adapted to its supersymmetric cousins because the BSFT for the superstrings have been developed [17, 18]. One can also generalize to include higher drivative terms not present in this work.

## 2. The model

Our approach is based on the old ideas of string coupling in background fields [13, 15] and the boundary string field theory formalism [19]–[21]: We couple some terms to the boundary of the string world-sheet, and calculate the partition function of the two dimensional sigma model for these fields. We then identify this partition functional of world-sheet couplings as a space-time action for these couplings, interpreted as fields. This approach was used originally to calculate the Born-Infeld action and string loop corrections for a gauge field, and was formalized later for massive and tachyonic boundary interactions.

We consider a toy model of a Dp-brane with world-sheet action on the disk

$$S = \frac{1}{4\pi\alpha'}\int_\Sigma \partial X^M\,\bar\partial X_M + \frac{c}{4\pi\alpha'}\int_{\partial\Sigma}\left(X^i - Y^i(X^\mu)\right)^2\,, \tag{2.1}$$

where $\Sigma$ and $\partial\Sigma$ are respectively the worldsheet and its boundary. The brane is parametrized in the static gauge by $p+1$ coordinates $X^\mu$, while being positioned at $Y^i$ in the remaining $d-p+1$ directions. Taking the limit $c\to\infty$ will ensure that the endpoint of the string are confined to move on the hypersurface defined by $Y^i(X^\mu)$. We use capital Roman indices $M, N$ to denote the total target space, if needed.

We compute the partition functional of $c$ and $Y^i$ through a euclidean path integral

$$Z(c, Y^i) = \int dX^i\,dX^\mu\,e^{-S}\,. \tag{2.2}$$

As was discussed in [20] for constant $Y^i$ there is a smooth interpolation between Neumann and Dirichlet boundary conditions for the coordinates $X^i$ as $c$ goes between 0 and $\infty$. In particular, the limit $c\to\infty$ is a conformally invariant point of the theory and we identify the partition functional in this limit with a spacetime effective action for the $Y^i$s. The general relation in bosonic BSFT is

$$S = (1+\beta_i\,\partial_i)\,Z\,, \tag{2.3}$$

where the derivatives are with respect to the boundary couplings and the $\beta$s are the corresponding beta-functions. In this case we consider the $c=\infty$ limit, which is a zero of the tachyon beta-function, and the fields $Y$ break the translational symmetry in the directions $X^i$ and so are massless. We then can identify

$$\lim_{c\to\infty} Z(c, Y^i) = S(Y^i)\,. \tag{2.4}$$

Prior to calculating the partition functional associated with the action (2.1) we review some useful facts concerning quadratic boundary interactions. For a single boson $X$ with boundary interaction $c\oint X^2$, the propagator for the oscillatory modes on the unit disk is

$$\langle X(z)\,X(z')\rangle = \frac{1}{2}\sum_{n=1}^{\infty}\frac{(z\,\bar z')^n + (\bar z\,z')^n}{n+2\,c}\,. \tag{2.5}$$

There is also an associated zero mode on the world sheet. One can show [20, 22] that the partition function for this boson (including the zero mode) is given by

$$\begin{aligned} Z &= \int dx\, e^{-cx^2} \prod_{n=1}^{\infty} \frac{1}{1+2\,c/n} \\ &= \int dx\, e^{-cx^2} \exp\left( \sum_k (-)^k \frac{\zeta(k)}{k} (2\,c)^k \right) \\ &= \int dx\, e^{-cx^2}\, e^{2ac}\, \Gamma(1+2\,c)\,. \end{aligned} \tag{2.6}$$

where $x$ denotes the zero mode of the string field. The parameter $a$ is an undetermined coefficient that regularizes the correlator $\langle X^2(0)\rangle$. Technically it is associated with the $\zeta$-function representation of the infinite product: $a = \zeta(1) + \gamma$ can assume different values depending on the regularization scheme used, however it does not affect physical quantities such as the ratio of brane tensions [22]. It is straightforward to generalize these results to multiple $X$'s with arbitrary quadratic interactions [23].

We now wish to write the boundary interaction term of (2.1) as a sum of a quadratic term and terms cubic and higher. We first write the string fields $X$ in terms of zero mode and oscillators:

$$X = x + \tilde{X}$$

and Taylor expand $Y(X^\mu)$ around the classical position of the brane, $Y^i(x)$, in powers of $\tilde{X}$:

$$Y^i(X^\mu) = Y^i(x^\mu) + \partial_\nu Y^i(x^\mu)\,\tilde{X}^\nu + \frac{1}{2}\partial_\nu\partial_\alpha Y^i(x^\mu)\tilde{X}^\nu\,\tilde{X}^\alpha + \cdots\,. \tag{2.7}$$

Having done this the interaction term of (2.1) becomes:

$$\begin{aligned} S_{int} &\equiv c\int_{\partial\Sigma} (x^i - Y^i)^2 + \tilde{X}^M U_{MN} \tilde{X}^N + \cdots \\ &= c\int_{\partial\Sigma} (x^i - Y^i)^2 + (\tilde{X}^i - \partial_\mu Y^i \tilde{X}^\mu - \frac{1}{2}\partial_{\mu\nu} Y^i \tilde{X}^\mu \tilde{X}^\nu)^2 + \\ &\quad + 2(x^i - Y^i)(\tilde{X}^i - \partial_\mu Y^i \tilde{X}^\mu - \frac{1}{2}\partial_{\mu\nu} Y^i \tilde{X}^\mu \tilde{X}^\nu) + \cdots\,, \end{aligned} \tag{2.8}$$

where we can read off $U$

$$U_{MN} = \begin{pmatrix} \delta^{ij} & -\partial_\nu Y^i \\ -\partial_\mu Y^j & \partial_\mu Y^i \partial_\nu Y^i - (x^i - Y^i)\partial_{\mu\nu} Y^i \end{pmatrix}. \tag{2.9}$$

We note that the boundary term linear in $\tilde{X}$ does not contribute to the partition function since the propagator (2.5) is periodic on the boundary of the unit disk. Furthermore since the propagator is proportional to $1/c$ in the limit $c \to \infty$, we can neglect cubic and higher boundary terms in the interaction because they give rise to $1/c$ corrections.

The form of the interaction calls for a field redefinition, $\tilde{X} \to J^{-1}\tilde{X}$:

$$\begin{aligned} \tilde{X}^i &\to \tilde{X}^i + \partial_\mu Y^i \tilde{X}^\mu \\ \tilde{X}^\mu &\to \tilde{X}^\mu - \partial_\mu Y^i \tilde{X}^i \end{aligned} \tag{2.10}$$

and

$$U \to J^T\, U\, J\,, \tag{2.11}$$

where $J^T$ is the transpose of $J$. Note also that the new metric

$$G = J J^T \tag{2.12}$$

is again block diagonal, while its longitudinal part coincides with the induced metric. Therefore the transformed fields $X^i$ are locally perpendicular and $X^\mu$ are locally parallel to the brane.

The above field redefinition gives rise to a jacobian for each oscillator mode. The jacobian is

$$\det\left(J^{-1}\right) = \begin{vmatrix} \delta^{ij} & -\partial^\mu Y^i \\ \partial^\nu Y^j & \delta^{\mu\nu} \end{vmatrix}\,, \tag{2.13}$$

so by $\zeta-$function regularization,

$$\prod_{n=1}^{\infty} \det J = (\det J)^{\zeta(0)} = (\det J)^{-1/2}\,. \tag{2.14}$$

We now follow [20, 22] and integrate out $\tilde{X}$ from (2.2) using the techniques outlined in (2.5) and (2.6). The partition functional is then naturally divided into contributions from the directions transverse to the world-volume, and the directions along the world-volume. The partition functional is

$$\begin{aligned} Z(c, Y^i) &= \int dx^M e^{-c(x^i - Y^i)^2} (\det J)^{-1/2} \exp\left( \sum_k (-)^k \frac{\zeta(k)}{k} (2\, c\, (UG)_{MN})^k \right) \\ &= Z(c) \int dx^M e^{-c(x^i - Y^i)^2} (\det J)^{-\frac{1}{2}} e^{2ac(UG)^\mu_\mu} \det \Gamma\left(1 + 2c(UG)_{\mu\nu}\right). \end{aligned} \tag{2.15}$$

We use $\mathrm{tr}(J^T U J)^k = \mathrm{tr}(UG)^k$, with $G$ defined by (2.12). In the above expression we have defined $Z(c)$ to be the partition function of the transverse modes.

We now wish to integrate out the (world-sheet) zero modes transverse to the brane. Our strategy will be to Taylor expand (2.15) in $x - Y$ and perform the gaussian integral order by order in this expansion. Since the function is strongly peaked around $x^i = Y^i$ the portion of $UG$ which depends on $x - Y$ is an appropriate small parameter. Motivated by desire to write the result in covariant form[1] we introduce the extrinsic curvature $K^i_{\mu\nu} = \partial_\mu \partial_\nu Y^i$.

[1] We recall a brief calculation in differential geometry. Consider an induced metric of the form

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \phi^i(x^\alpha) \partial_\nu \phi^i(x^\alpha)\,.$$

The Ricci tensor for this metric is

$$R_{\nu\beta} = g^{\mu\alpha} \left( \partial_\mu \partial_\alpha \phi^i \partial_\nu \partial_\beta \phi^j - \partial_\alpha \partial_\beta \phi^i \partial_\mu \partial_\nu \phi^j \right) \left( \delta^{ij} - g^{\delta\gamma} \partial_\delta \phi^j \partial_\gamma \phi^i \right).$$

Similarly, with the index $i$ perpendicular to the $\mu, \nu$, the extrinsic curvature is

$$K^i_{\mu\nu} = \partial_\mu \partial_\nu \phi^i\,.$$

When we Taylor expand (2.15) in $(x^i - Y^i)K^i_{\mu\nu}$ we obtain

$$
\begin{aligned}
Z(c,Y) = Z(c) \int dx^M e^{-c(x^i-Y^i)^2} (\det J)^{-1/2} \times \\
\times \Big\{ 1 + 2\Big( a^2 \,\mathrm{tr}\, K^i \,\mathrm{tr}\, K^j + \zeta(2)\,\mathrm{tr}\, K^i K^j \Big)(x^i - Y^i)(x^j - Y^j) + \\
+ \frac{2}{3}\Big( a^4 \,\mathrm{tr}\, K^i \,\mathrm{tr}\, K^j \,\mathrm{tr}\, K^l \,\mathrm{tr}\, K^m + 6a^2\zeta(2)\,\mathrm{tr}\, K^i \,\mathrm{tr}\, K^j \,\mathrm{tr}\, K^l K^m + \\
+ 6\zeta(4)\,\mathrm{tr}\, K^i K^j K^l K^m - 8a\zeta(3)\,\mathrm{tr}\, K^i \,\mathrm{tr}\, K^j K^l K^m + \\
+ 3\zeta(2)^2 \,\mathrm{tr}(K^i K^j)\,\mathrm{tr}(K^i K^j) \Big) \times \\
\times (x^i - Y^i)(x^j - Y^j)(x^l - Y^l)(x^m - Y^m) + \cdots \Big\}. \qquad (2.16)
\end{aligned}
$$

We note that the choice of $a^2 = -\zeta(2)$ results in the $O(\alpha')$ terms being expressed in a simple way, however, this corresponds to setting the $z \to z'$ limit of (2.5) to be $i\sqrt{\zeta(2)}$, a somewhat non-standard regularization. We restore the dependence on $\alpha'$, and simultaneously note that there is no longer any dependence on the parameter $c$, at every order in the expansion below. We perform the integrations for the terms both quadratic and quartic in $x^i - Y^i$ and obtain an effective action for the fluctuations $Y^i$,

$$
\begin{aligned}
S(Y) = \mathrm{Z(c)} \int dx^\mu \sqrt{\det(G_{\mu\nu})} \Big( 1 - \alpha'\zeta(2)R + \alpha'^2\zeta(2)^2 \frac{1}{2} R^2 + \\
+ \alpha'^2 \zeta(2)^2 \left( -2\,\mathrm{tr}\, K^i K^j \,\mathrm{tr}\, K^i \,\mathrm{tr}\, K^j + \mathrm{tr}\, K^i K^j \,\mathrm{tr}\, K^i K^j \right) + \\
+ \alpha'^2 \left( -i4\sqrt{\zeta(2)}\zeta(3) \right) \mathrm{tr}\, K^i \,\mathrm{tr}\, K^i K^j K^j + \\
+ \alpha'^2 \zeta(4) \left( 2\,\mathrm{tr}\, K^i K^i K^j K^j + \mathrm{tr}\, K^i K^j K^i K^j \right) \cdots \Big). \qquad (2.17)
\end{aligned}
$$

All inner products are with respect to the metric $G_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu Y^i \partial_\nu Y^i$. When we compute (2.17) we make the identification $(\det J_{MN})^{-1/2} = \sqrt{\det G_{\mu\nu}}$ as can be easily verified. The terms at order $\alpha'^2$ do not either obviously cancel, or combine into a Gauss-Bonnet term $R^2 + 4R^2_{AB} + R^2_{ABCD}$, although it is certainly possible that some field redefinition could combine these terms. We can argue that the imaginary part (or more weakly the presence of a quadratic term in $R$ indicating that the ghosts do not decouple) is a reflection of the well-known instability for bosonic branes.

It is also possible, and perhaps more democratic, to write (2.17) solely in terms of the extrinsic curvature. We may additionally discard the indeterminate terms involving $a$, in line with the regularization adopted in [19]:

$$
\begin{aligned}
S(Y) = \int dx^\mu \sqrt{\det(G_{\mu\nu})} \Big[ 1 + \alpha' \frac{\pi^2}{6} \,\mathrm{tr}\, K^i K^i + \\
+ \alpha'^2 \frac{\pi^4}{18} \Big( \frac{1}{5} \left( 2\,\mathrm{tr}\, K^i K^i K^j K^j + \mathrm{tr}\, K^i K^j K^i K^j \right) + \qquad (2.18) \\
+ \frac{1}{4} \left( 2\,\mathrm{tr}\, K^i K^j \,\mathrm{tr}\, K^i K^j + \mathrm{tr}\, K^i K^i \,\mathrm{tr}\, K^j K^j \right) \Big) + \cdots \Big].
\end{aligned}
$$

As an exercise, it is possible to add the coupling to a U(1) gauge field to this model. To effect this, we add the coupling

$$\delta S = \oint F_{\mu\nu} X^\mu \partial_t X^\nu \tag{2.19}$$

to the action (2.1). It is very easy to verify that this addition changes the action (2.17) in the following ways: First the prefactor which was the square root of the determinant of the metric becomes the Born-Infeld term,

$$\sqrt{\det G} \to \sqrt{\det G + 2\alpha' F}\,, \tag{2.20}$$

and second, the metric used in the contractions of $R$ and $K$ gains a non-diagonal term

$$G^{\mu\nu} = G^{\mu\alpha} \left( \frac{1}{1 + 2\alpha' F_{\alpha\beta} G^{\beta\nu}} \right)_\alpha^{\ \nu} . \tag{2.21}$$

An alternative way to derive all the results discussed here is to start with action (2.1), then Taylor expand the interaction term as in (2.8). Next impose that the $X^i$ coordinates have Dirichlet boundary conditions, and treat the $\partial_\mu Y^i \partial_\nu Y^i$ term as a boundary mass term. Calculations exact in this, and perturbative in $(x^i - Y^i)\partial_\mu \partial_\nu Y^i$ give the same results as those discussed above.

We have given this argument in the simplest possible case, namely we take the limit where $c \to \infty$. Using the boundary string field theory technology it is possible to generalize this construction to the case of large but finite $c$. This would describe branes of thickness $\propto 1/\sqrt{c}$ which have been of interest (see for example [24]). Also, we have only considered the cases of a bosonic brane with or without a U(1) gauge field, and the generalization to other types of matter is of interest.

## 3. Discussion

We now summarize the steps that we took in the derivation of this result. We started by modelling a hypersurface by a confining quadratic potential for the ends of the open string. The classical position of this surface depended on the transverse coordinates, and the limit where the strength of the potential went to infinity corresponded to Dirichlet boundary conditions for the open string, and hence the surface was identified as a D-brane. We then taylor-expanded the boundary interaction term, keeping only the quadratic part of the boundary potential, since the cubic and higher order terms gave corrections that vanished in the limit we consider. A change of variables so that the $X$ fields were locally tangential or normal to the brane was performed, and the jacobian for this transformation was identified with $\sqrt{\det(\eta + \partial Y \partial Y)}$ exactly as we expect for the induced gravity. Then we derived the partition function for the system, and taylor expanded in the zero mode term, finally integrated out the transverse zero modes. This procedure gives us the $\alpha'$ expansion. We identified the result of this integration as the space-time effective action for the brane, and expressed it in terms of curvature invariants on the brane.



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The normal ordering prescription that was used to obtain (2.17) may not be the most natural. We expect that this ambiguity in $a$ will be eliminated in superstrings.[2] Expressing the action for the transverse scalars in terms of the extrinsic curvature $K$ may make the resulting world-volume action more amenable to supersymmetrization.[3] The physics of a brane model with gravity built from the extrinsic curvature remains an open problem. Other researchers have considered theories with a maximum acceleration and found that the generic lagrangian in those cases is a power series in the trace of the Riemann tensor [27]. The present simple model does not give rise to such a class of theories. However the nice form into which the extrinsic and intrinsic curvature have organized themselves calls for further explanation. Finally it is very interesting to study the possible implications of this modified action to newtonian gravity.


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